G 423 (Pages : 4) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, APRIL 2010

Second Semester

Core Course: ANALYTIC GEOMETRY, TRIGONOMETRY AND MATRICES

(Common for B.Sc. Mathematics Model-I, B.Sc. Mathematics Model-II and B.Sc. Computer Applications [Three Main]

Time : Three Hours Total Weightage : 25

Part A (Objective Type Questions)

Answer **all** *the questions.*
A bunch of 4 questions has weight 1.

I. 1 Write a parametric representation of a point on the parabola $y^2 = \frac{-x}{2}$.

2 What is the slope of the tangent at 't' on the parabola $y^2 = 4ax$?

3 State the condition for which the lines $y = mx$ and $y = m_1 x$ are conjugate diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

4 What is the diameter of the auxiliary circle of the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$?

II. 5 What is the general equation of a rectangular hyperbola ?

6 What is the angle between the asymptotes of a hyperbola if its eccentricity is e ?

7 Write the equation of the hyperbola conjugate to $\frac{x^2}{4^2} - \frac{y^2}{8^2} = 1$.

8 Give an equation of a line perpendicular to $p = r\cos(\theta - \alpha)$ in polar co-ordinate system.

III. 9 What is the value of e if $\frac{l}{r} = 1 + e\cos\theta$ is a circle ?

10 What is the relation between $\sin^2(iy)$ and $\sin h^2 y$?

11 What is the period of the hyperbolic cosine function ?

12 What is the real part of $\cosh(\alpha + \beta i)$?

Turn over

IV. 13. What is the rank of the matrix ?

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & -3 & 6 \\ -8 & 8 & -16 \end{bmatrix}.$$

14. If a system of 'n' linear equations in 'n' unknowns have a solution other than the trivial solution, then what can we say about rank of A ?

15. What are the characteristic roots of $\begin{bmatrix} -1 & 2 \\ 0 & 8 \end{bmatrix}$?

16. Write a matrix with characteristic roots 2 and –3.

(4 × 1 = 4 weight)

Part B (Short Answer Type Questions)

Answer any **five** *questions.*
Each question has weight 1.

17. Find the equation of the normal at a point 't' on the parabola $y^2 = 4ax$ using the equation of the tangent at 't'.

18. Prove that the sum of the squares of two conjugate semi-diameters of an ellipse is constant.

19. Prove that if the feet of the normals from (h, k) to the rectangular hyperbola $xy = c^2$ are t_1, t_2, t_3 and t_4, then $t_1 t_2 t_3 t_4 = -1$.

20. Convert the equation of the line $Ax + By + C = 0$ from Cartesian co-ordinate system to polar co-ordinate system.

21. Prove that $\cosh^2 y - \sinh^2 y = 1$ for all y.

22. Prove that $\sinh 3x = 3\sinh x + 4\sinh^3 x$.

23. Find all solutions of $x_1 - 2x_2 + x_3 - 3x_4 = 1$.

24. Find the characteristic polynomial of $\begin{bmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \\ 3 & -3 & 6 \end{bmatrix}$.

(5 × 1 = 5 weight)

Part C (Short Essay Type Questions)

Answer any **four** *questions.*
Each question has weight 2.

25. Show that the condition for the line $lx + my + n = 0$ to be a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

$$\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{\left(a^2 - b^2\right)^2}{n^2}.$$

26. Find the asymptotes of the hyperbola :

$$3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0.$$

27. Give the co-ordinates of the foci and the vertices and the equations to the axes of the conic

$$\frac{3}{r} = 1 + 2\sin\theta.$$

28. Resolve $x^6 + 1$ into factors.

29. Write the normal form of :

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \end{bmatrix}.$$

30. Find all non-trivial solutions of :

$$\begin{aligned} 2x_1 - x_2 + 3x_3 &= 0 \\ 3x_1 + 2x_2 + x_3 &= 0 \\ x_1 - 4x_2 + 5x_3 &= 0. \end{aligned}$$

$(4 \times 2 = 8$ weight$)$

Part D (Essay Type Questions)

Answer any **two** *questions.*
Each question has weight 4.

31. P is a point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and Q is the corresponding point on the auxiliary circle. If the normals at P and Q meet in R, show that the locus of R is $x^2 + y^2 = (a + b)^2$.

Turn over

32. Sum the series :

$$\sin\alpha + \frac{1}{2}\sin 2\alpha + \frac{1}{2^2}\sin 3\alpha + \ldots \text{ to infinity.}$$

33. Find the characteristic roots and the associated characteristic vectors of the matrix :

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}.$$

(2 × 4 = 8 weight)